SPH (smoothed-particle hydrodynamics)

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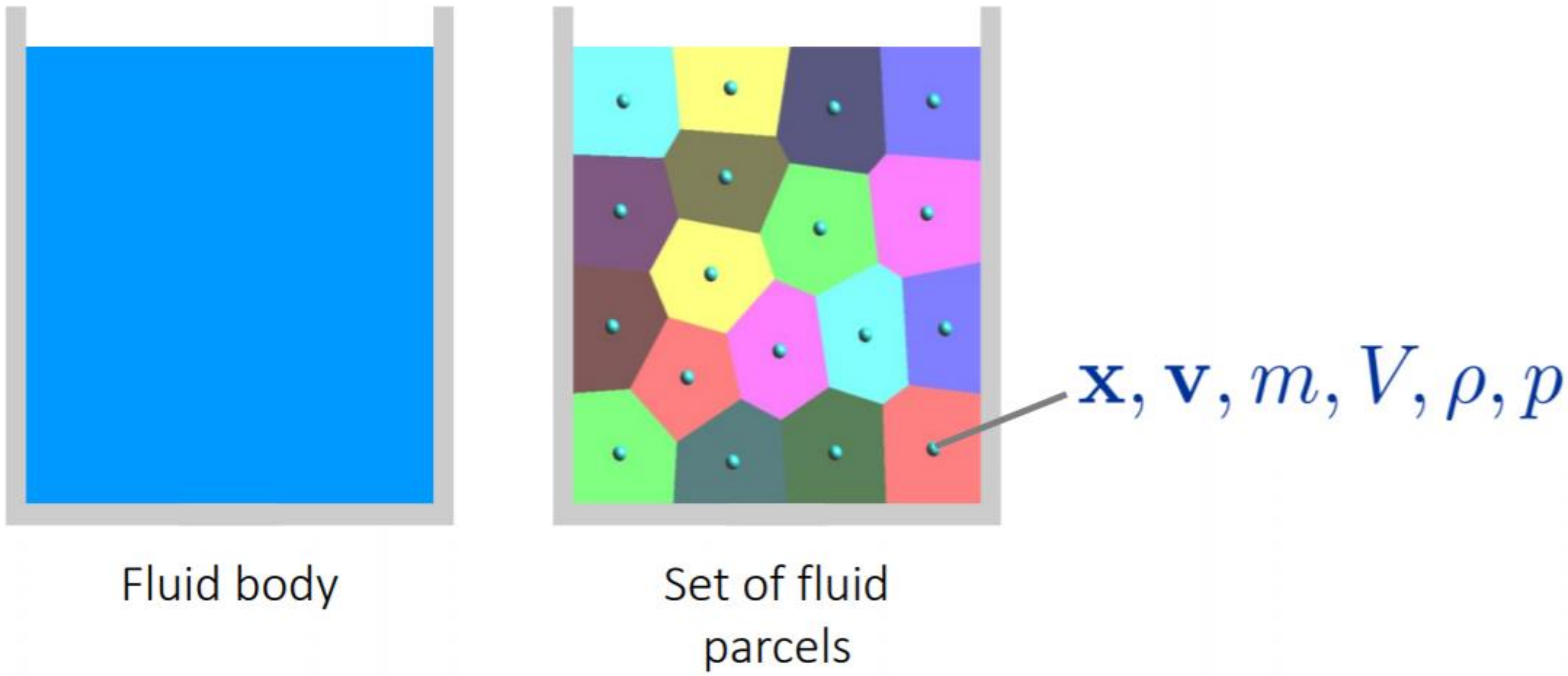
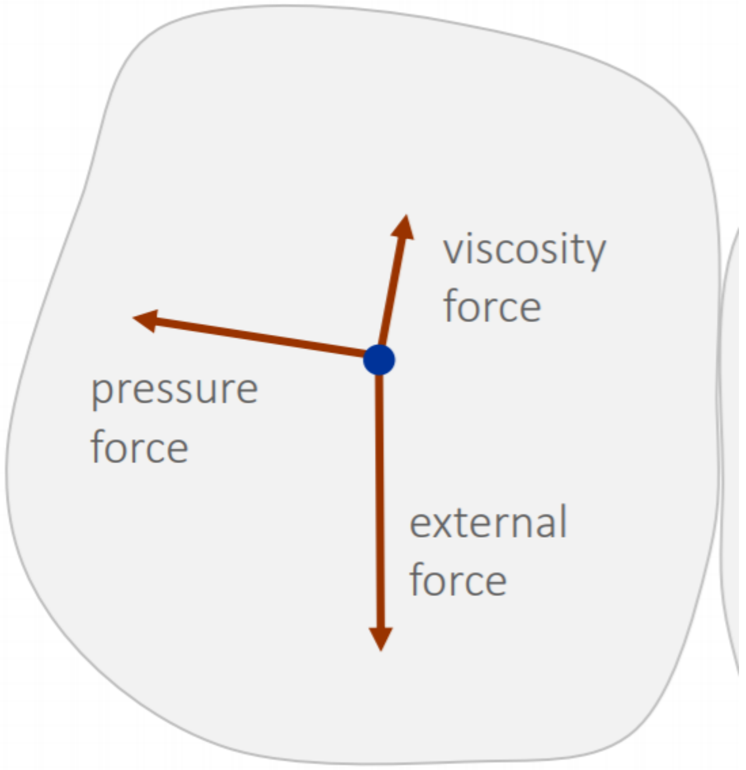
I use this to show my basic understanding of SPH

First, we make fluid it into particles



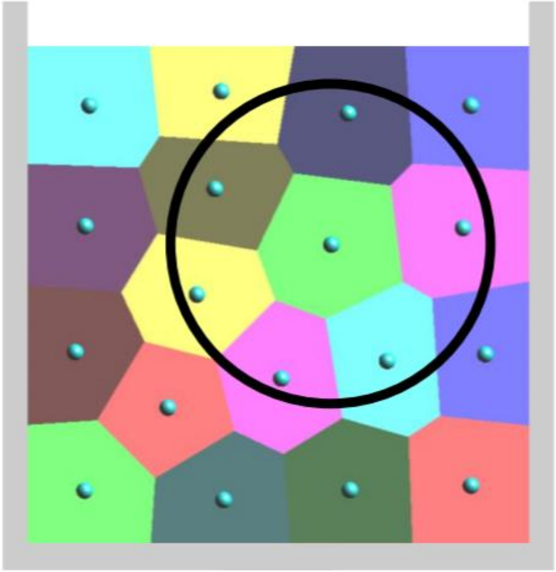
SPH follows Newton's Rule

- 3 Forces in total:
- Gravity F: ρg (external F)
- Pressure F: $-\nabla p$
- Viscosity F: $\mu \nabla^2 v(t)$



- Acceleration: $\sum a = \frac{\sum F}{\rho}$

Kernel Function



- Basically, we use kernel f to calculate the density, pressure, and viscosity of particle in r(i) based on the influences of other particles in a certain range.
- Basic formula: $A(\vec{r}) = \sum_j A_j \frac{m_j}{\rho_j} W(\vec{r} - \vec{r_j}, h)$
- (A: the quantity calculated; W: Kernel weights of particle j; h defines the furthest r(j), i.e., if $|r(i) - r(j)| \geqslant h$, W = 0)

Kernel Function

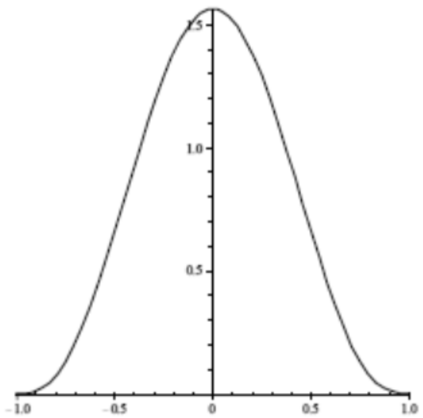
- Based on the three equations below, we can get corresponding quantities we want for particle i.

$$\rho(r_i) = \sum_j \rho_j \frac{m_j}{\rho_j} W(\vec{r_i} - \vec{r_j}, h) = \sum_j m_j W(\vec{r_i} - \vec{r_j}, h) \quad (1)$$

$$\vec{F_i}^{pressure} = -\nabla p(\vec{r_i}) = -\sum_j p_j \frac{m_j}{\rho_j} \nabla W(\vec{r_i} - \vec{r_j}, h) \quad (2)$$

$$\vec{F_i}^{viscosity} = \mu \nabla^2 \vec{u}(r_i) = \mu \sum_j \vec{u_j} \frac{m_j}{\rho_j} \nabla^2 W(\vec{r_i} - \vec{r_j}, h) \quad (3)$$

Here u means velocity V



Kernel Function:
Gaussian-like Function

Basic algorithms

1. We randomly set a series particles.
2. By applying formula (1) in last page, we can get density
3. By applying formula (2)&(3), we can get F(pressure), F(viscosity), and F(gravity).
4. Get the a and by applying formula below, we can update the system.

$$\mathbf{v}_i(t+\Delta t) = \mathbf{v}_i(t) + \Delta t \mathbf{a}_i(t)$$
$$\mathbf{x}_i(t+\Delta t) = \mathbf{x}_i(t) + \Delta t \mathbf{v}_i(t+\Delta t)$$

5. Repeat steps 1-4 and do iterations.