Energy Autocorrelation Function

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October 2021

1 Autocovariance

e_t represents the energy of the system at time t, and by time interval k, or lag k, we get energy e_{t+k}. Now we estimated define covariance $\hat{\gamma}_k$ with lag k under non-infinite observations N as $cov[e_t, e_{e+k}]$, which is [1]:

$$\hat{\gamma}_k = \frac{1}{N}\sum_{t=1}^{N-k}(e_t - \mu)(e_{t+k} - \mu) \tag{1}$$

where μ is the average energy and $k = 0, 1, 2, 3, ..., K$. Here we set K not larger than N/1000.

2 Autocorrelation

Then, autocorrelation function at lag $k$is defined as [1]:

$$\hat{\rho}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0} \tag{2}$$

2.1 Matlab Code

```
function [ACF] = getACF(x1,lagMax)
x = x1 -mean(x1);
n = length(x);
ACF = zeros(1,lagMax+1);
ACF_normalize = zeros(1,lagMax+1);
for tau = 0:lagMax
        ACF(tau+1) = nansum(x .* lagmatrix(x,-tau)) / n;
end
ACF = ACF/ACF(1);
end
```

3 Results Compared with SIF

The results are shown in Fig.1.

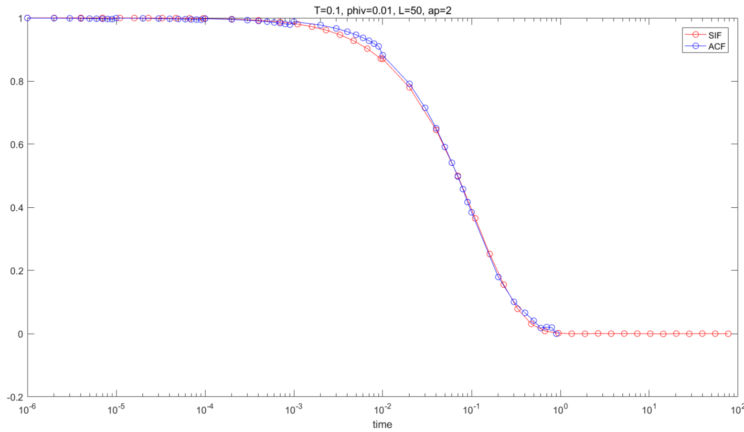

Figure 1: Results of energy autocorrelation function (ACF) and self-intermediate function (SIF) of a DPLM system with parameters shown in the title. Noted that here ACF curve is drawn by ACF results from 5 different time-scale simulations (with other parameters same), and for each result we extract their first ten time unit results, i.e., first ten lags ($time = dt * lags$). For example, if time scale is from $(10^{-6} - 10^{-1})$, we extract a $(10^{-6} - 10^{-5})$ result, and with five extracted results from $(10^{-6} - 10^{-5})$ $(10^{-5} - 10^{-4})$ ... $(10^{-1} - 10^{-0})$, we get the blue curve.

References

[1] G. E. P. Box, G. M. Jenkins, and G. C. Reinsel, "Time series analysis: forecasting and control," *Hoboken, New Jersey : John Wiley Sons, Inc.*, pp. 24–25, 2016.